Tunnel Junctions

In this article, Dr Benoît Jones looks at tunnel junctions and how to design them.

Although it is easier to design nice straight bits of tunnel, unfortunately for practical reasons we often need lots of junctions to join tunnels together. These occur whenever two or more tunnels intersect, so there may be at least 20-30 junctions in a typical metro station, and even road and rail running tunnels need vent tunnels, shafts, drainage sumps and crosspassages, all of which involve intersecting cylinders.

We perhaps know that we are likely to get stress concentrations around tunnel junctions, and that these will require strengthening of the lining, but in order to design the strengthening we need to be able to quantify the stress concentrations.

Simplified 2D plane stress models

Usually a larger diameter 'parent' tunnel or shaft is constructed first and then a hole is created in the lining and a smaller 'child' tunnel (or shaft) is constructed from it. This means that the critical stage is when the hole is made for the breakout from the parent tunnel. The hoop stresses have to divert around the hole, resulting in higher hoop stresses to either side and tension above and below in the longitudinal direction (Figure 1).

Therefore, a very simplified method of calculating stresses around junctions would be to envisage the parent tunnel or shaft as a curved plate with a hole in it. If this curved plate were then flattened we would have an even simpler geometry: a 2D plate with a hole in it.

Figure 1: Hoop stress in (a) a straight parent tunnel (b) parent tunnel after breakout for a child tunnel

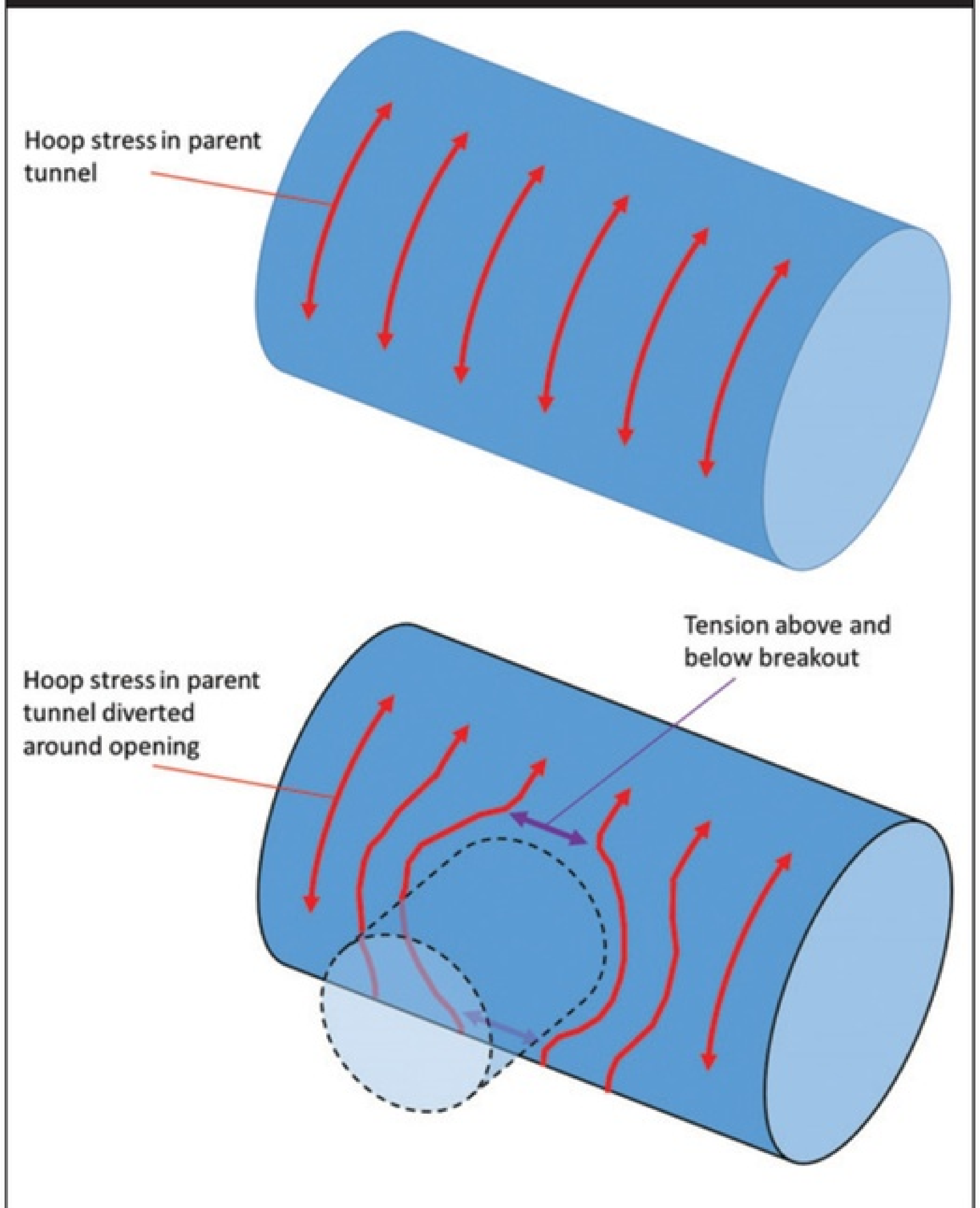
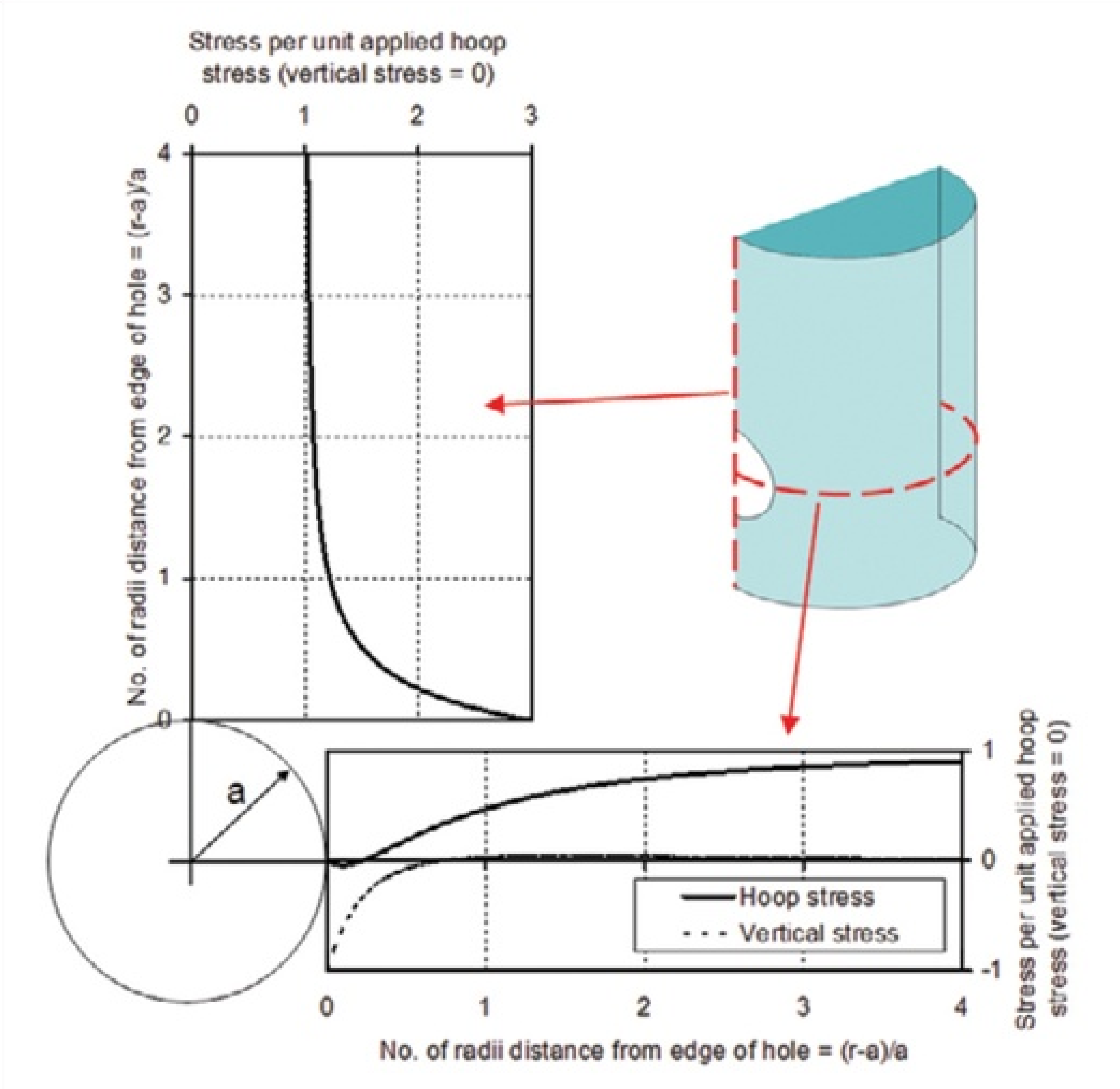


Figure 2: Stress concentrations at opening axis level and along a vertical line above and below an opening in a shaft from the Kirsch solution for an infinite elastic plane stress plate with a hole



Fortunately, there exists an exact analytical solution for a 2D plane stress plate with a hole, attributed to Kirsch in 1898 (the equations may be found in Hoek & Brown, 1980). 'Plane stress' means that there is only stress within the plane of the plate. Another way of saying this is that there is no stress in the out-of-plane direction, but there can be strain. This is different to plane strain, where there is no strain in the out-of-plane direction, but there can be stress. It is important not to get these confused. The plate is also assumed to be infinite and elastic.

The Kirsch solution tells us that hoop stress is increased by a factor of 3 either side of the opening, and that the tension above and below the opening in Figure 1(b) is equal to the initial hoop stress (but with opposite sign, i.e. tension rather than compression).

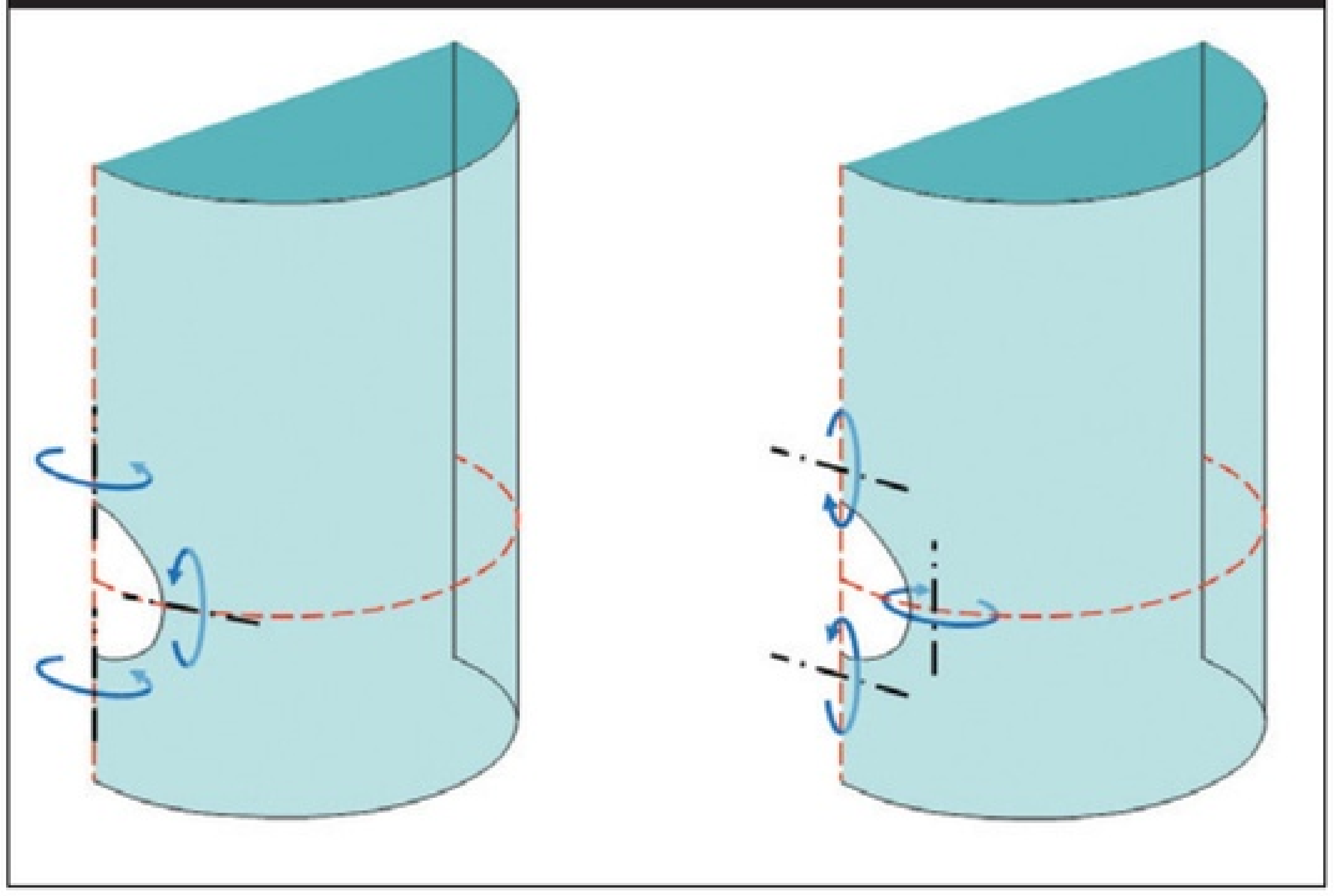
If we now take a shaft and unroll the lining so it is still standing up but is flat like a billboard, then the hoop stress due to ground load is in the horizontal direction. For simplicity let's assume the stress in the vertical direction is zero. Then if we cut a hole in it, the Kirsch solution will give us the stress concentrations shown in Figure 2. Similar stress concentrations would be found around an opening in a tunnel – just rotate Figure 2 (or your head) by 90°.

Because we have unrolled the lining into a 2D flat surface, we are not including the effect of the curvature of the lining on the stress concentrations. This is because a 2D plane stress plate has no bending stresses, but we know in reality we will get significant bending stresses in the lining around an opening. For the example of an opening in a shaft, we know that at the axis level of the opening the shaft lining will try to close in. In turn this will cause the lining above and below the opening to push out. This can be best demonstrated by cutting a hole in a toilet roll and then squeezing it. I have attempted to draw these moments in Figure 3; at each location there are moments about two axes but I have drawn these on separate diagrams so that they can be seen more easily.

Simple 'structural' 3D models

The only way to try to predict bending stresses as well as axial stresses around an opening would be to model the lining in a 3D numerical model. The simplest 3D numerical model is one where only the lining is modelled, and the ground loads are applied using an external pressure. However, there are several problems with these simple models.

Figure 3: Moments about different axes near an opening



1. Usually, the parent and child tunnel are modelled as a finished product, so the construction sequence is not included. This means that the stress concentrations around the junction are shared across the parent and child tunnel, whereas in reality the stresses are redistributed when the hole is made in the parent tunnel and the child tunnel is only constructed afterwards.
2. If an external applied pressure is used to simulate the ground loads, there is no soil-structure interaction. There are ways to create a pseudo-soil-structure interaction using springs, so that as the structure moves away from the ground the loading on the structure is reduced and vice-versa. However, redistribution of stresses from one area of ground to another is not possible because the springs are independent.
3. Another missing feature is that when the opening is made in the parent tunnel lining, the ground will deform towards the opening, and arching around the opening will result in higher ground pressures on the outside of the lining around the edges of the hole. Using a simple 3D numerical model means that there is no redistribution of this type. Even if the construction sequence is modelled, the applied pressure on the lining before breakout is effectively deleted when the lining is deleted.

Having said all this, there are occasions where this kind of model may be perfectly adequate. One such situation is where the majority of the loading is applied after construction. This may occur in a pressure tunnel (e.g. for hydroelectric), or where a secondary lining is being designed mainly for long-term ground loads and external water pressure.

Full 3D models

To summarise what has been discussed so far:

- the Kirsch solution gives only an approximation of axial stresses, but doesn't calculate bending stresses
- simple structural 3D numerical models predict bending stresses, but because construction sequence and soil-structure interaction are not modelled, the answers are not just inaccurate but often plain wrong.

The only solution left, therefore, is to design junctions in 3D with the ground and the construction sequence modelled explicitly. An example of such a model is shown in Figure 4.

In this case the ground was modelled by solid elements and included small strain stiffness and a Mohr-Coulomb failure criterion. The lining was modelled by linear elastic shell elements.

Figure 4: Model of a shaft-tunnel junction (Jones, 2007)

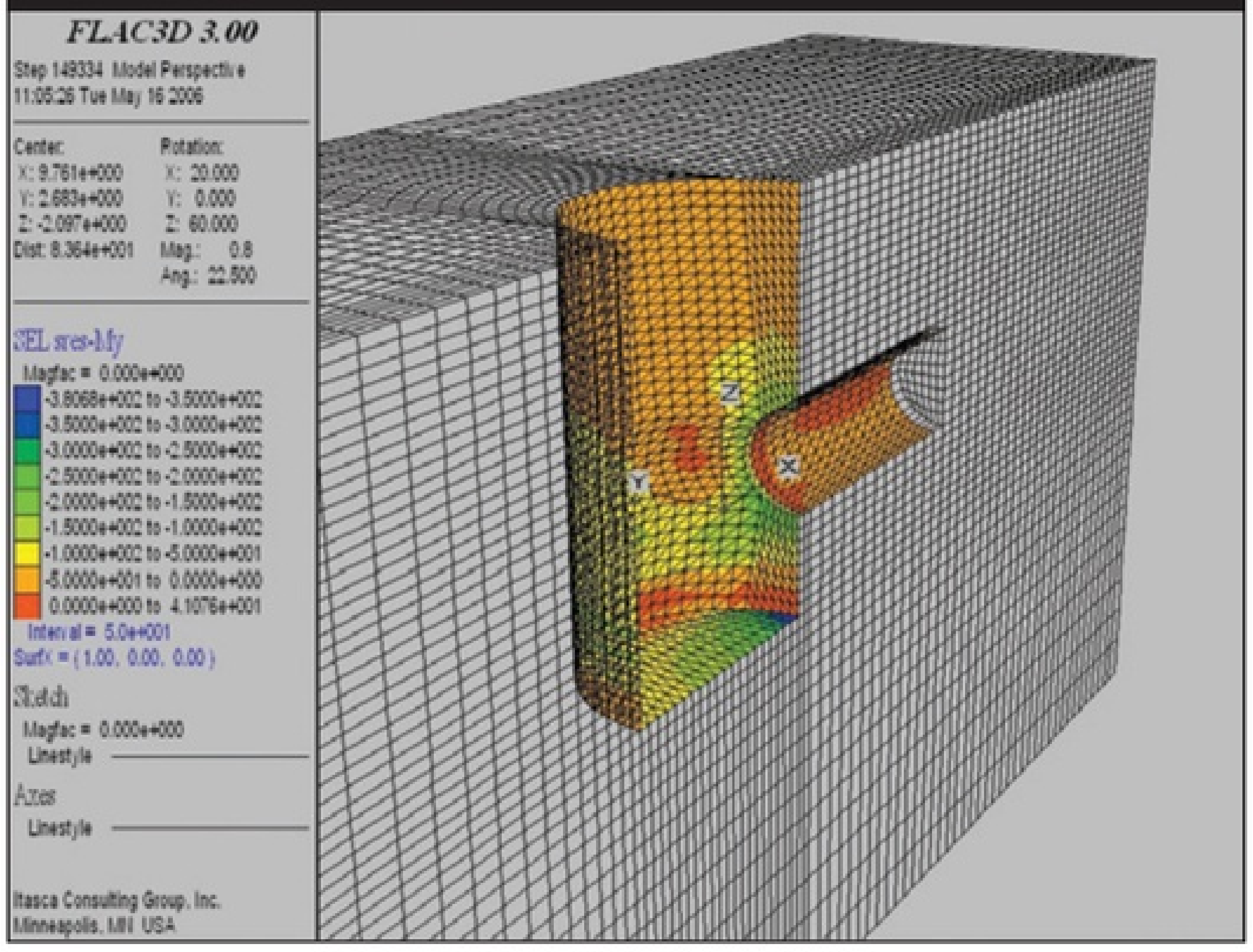
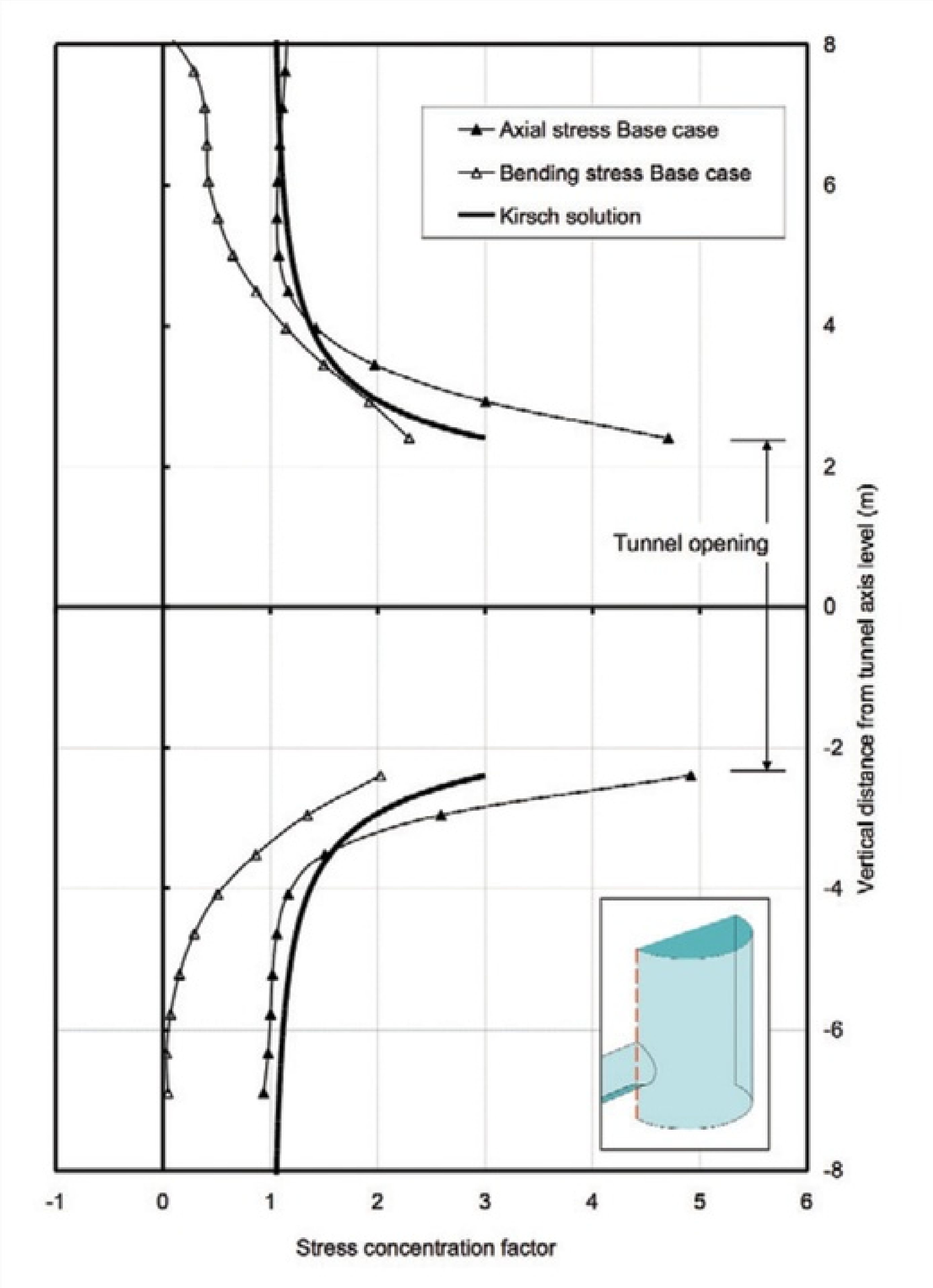


Figure 5: Comparison of 3D numerical model with construction sequence and ground modelled explicitly with Kirsch solution (Jones, 2007). Stresses shown are in the hoop direction along a vertical line through the centre of the opening.



Figures 5, 6 and 7 show the stresses in the shaft lining when the opening has just been made but the child tunnel construction has not yet begun. The pattern of axial stress is similar to the Kirsch solution, but the magnitudes are not. Also evident are the bending stresses, which are not

Figure 6: Comparison of 3D numerical model with construction sequence and ground modelled explicitly with Kirsch solution (Jones, 2007). Stresses shown are in the hoop direction along a horizontal line through the centre of the opening

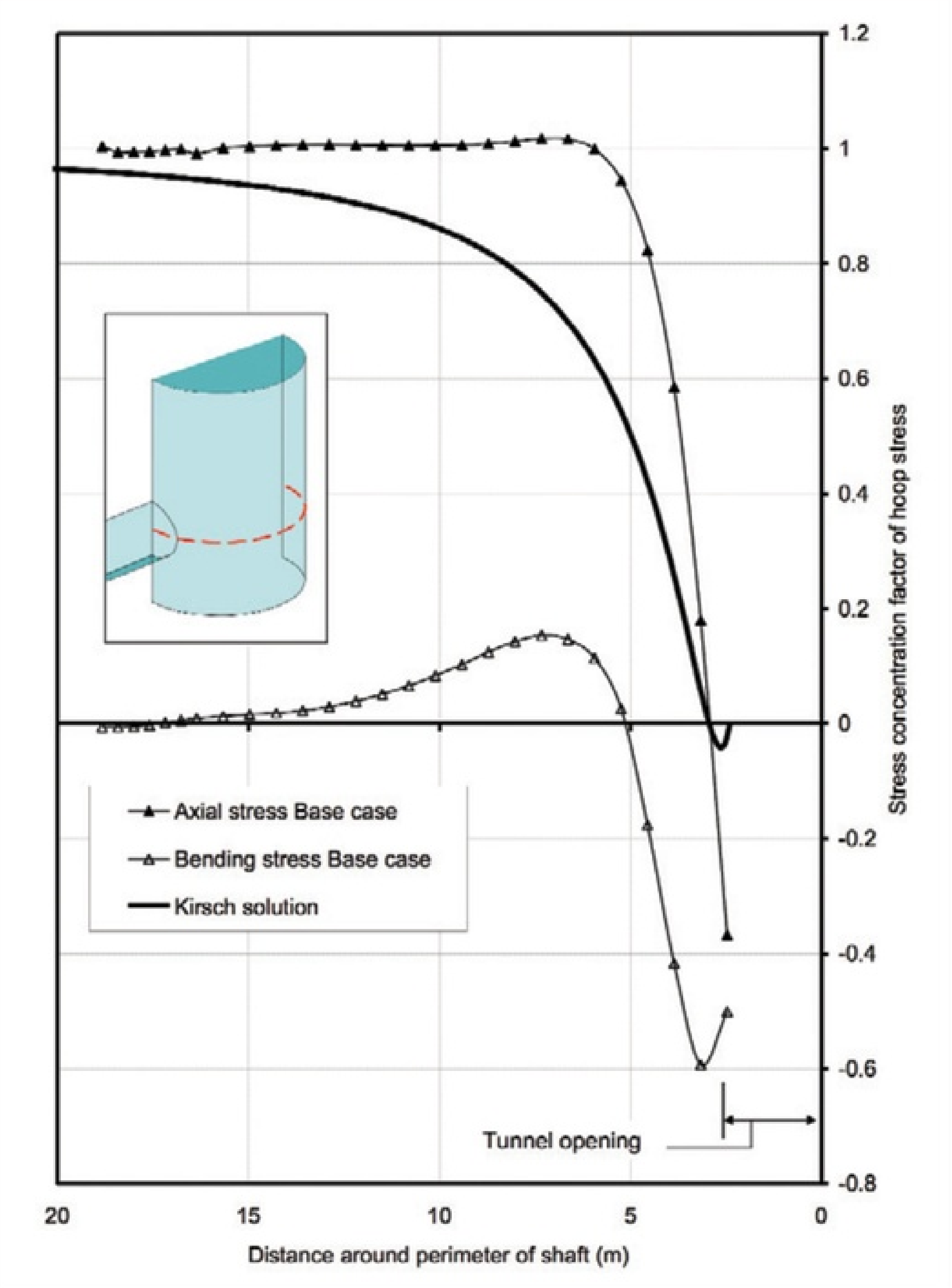
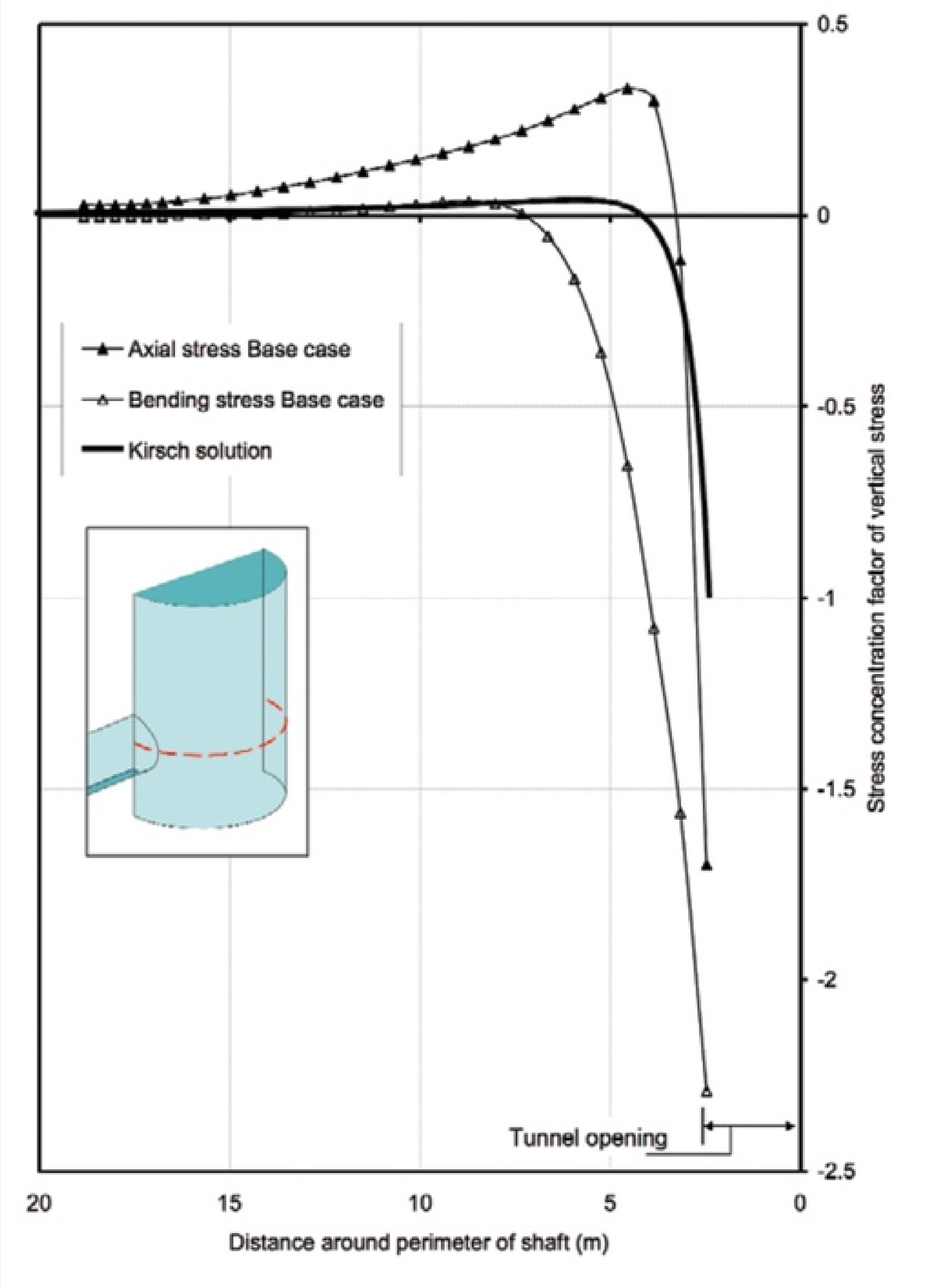


Figure 7: Comparison of 3D numerical model with construction sequence and ground modelled explicitly with Kirsch solution (Jones, 2007). Stresses shown are in the vertical direction along a horizontal line through the centre of the opening.



provided by the Kirsch solution but are certainly not negligible.

In Figure 5, the hoop stress after the opening is created is nearly 5x larger than it was before. The Kirsch solution predicted the maximum stress concentration should be 3x. This may be due to stress redistribution in the ground, and soil-structure interaction. As the opening is formed, the ground will deform towards and arch around the opening, increasing the ground pressure onto the relatively rigid shaft lining. Also, as mentioned before, above and below the opening the lining will experience a hogging moment and push into the ground; this should be expected to further increase the applied ground pressure at this location.

Figure 5 also shows the bending stress in the hoop direction (i.e. about a vertical axis) as a multiple of the hoop axial stress prior to breakout. These values can be added to or subtracted from the axial stress to give the stress at the intrados or extrados. The maximum compressive stress is therefore about 7x the hoop stress prior to breakout.

In Figure 6 the 3D numerical model is showing tensile axial stress in the hoop direction close to the edge of the opening. This may be because of ground movements towards the opening dragging on the lining, and stretching caused by the 3D deformation. Another difference is that compared to the Kirsch solution for a 2D plane stress plate, the hoop stress in the numerical model drops more dramatically closer to the opening. This may be the curvature of the lining causing the diversion of hoop stresses around the opening to be more localised. Again, the bending stresses are far from negligible and will increase requirements for reinforcement or strengthening.

In Figure 7, the vertical axial stresses to the sides of the opening are higher in the numerical model compared to the Kirsch solution. This may be due to the more sudden and localised diversion of hoop stresses around the opening discussed in the previous paragraph, perhaps coupled with stretching due to 3D deformation. The bending stresses in the vertical direction (i.e. bending about a horizontal axis) are very large at this location and will contribute greatly to requirements for reinforcement or strengthening.

Conclusions

The behaviour of tunnel junctions, and the merits of various design methods, have been discussed. Although the Kirsch solution provides an understanding of the axial stress concentrations around a junction, because it is in 2D it does not predict the bending stresses, which can be very significant. Simple 'structural' 3D numerical models provide bending moments, but except in cases where the loading is applied after construction, the pattern and magnitude of the bending moments are not just inaccurate, but wrong as they fail to model soil-structure interaction or the construction sequence. The only approach with a hope of getting close to the correct answer is 3D numerical modelling with the construction sequence and the ground modelled explicitly.

REFERENCES

Hoek, E. & Brown, E. T. (1980). Underground Excavations in Rock, 527pp. London: IMM.

Jones, B. D. (2007). Stresses in sprayed concrete tunnel junctions. EngD thesis, University of Southampton, Faculty of Science, Engineering and Mathematics.