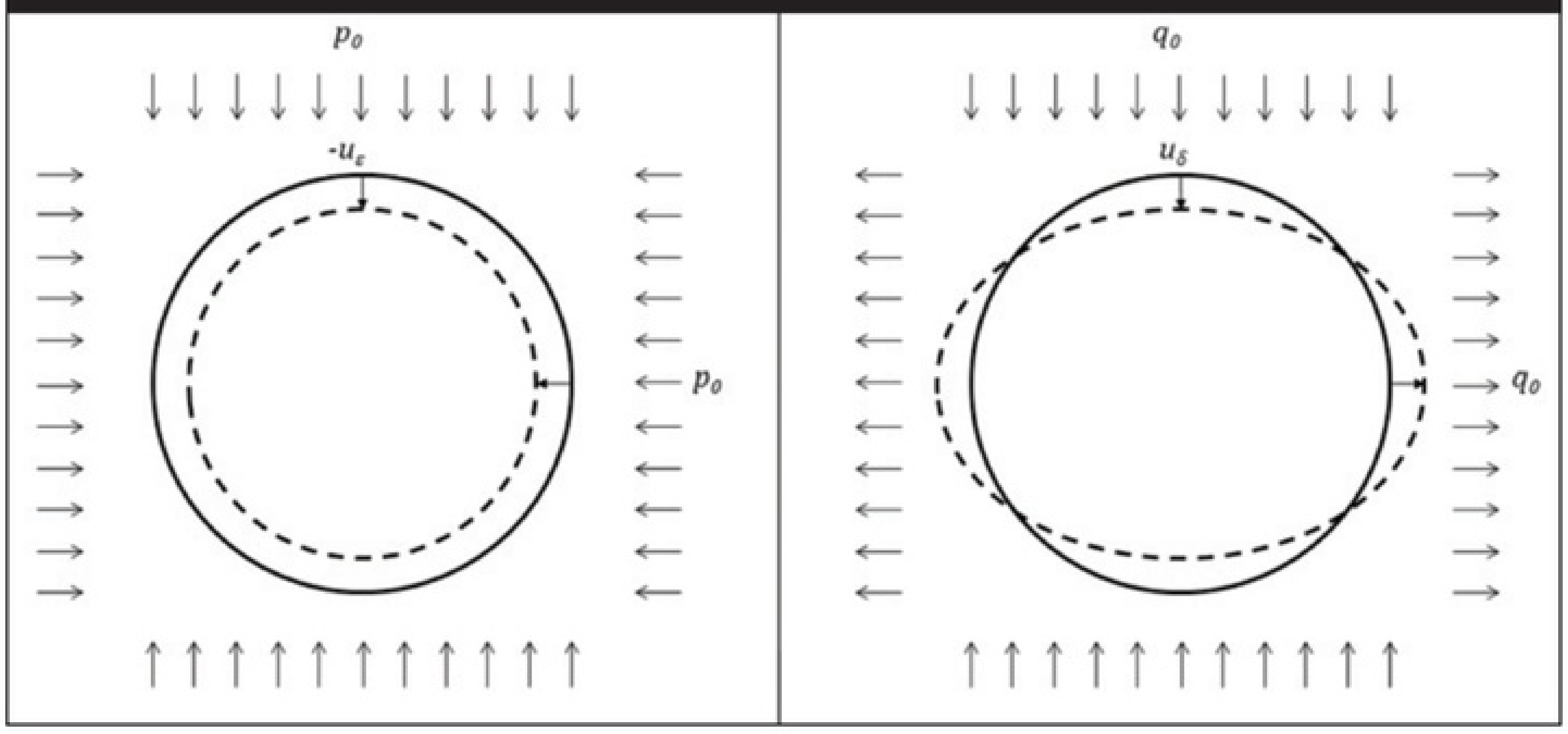
In this article, Dr Benoît Jones, Director of the Tunnelling and Underground Space MSc at the University of Warwick, UK, looks at methods of estimating horizontal ground movements due to tunnelling.

Horizontal ground movements

WHEN EXCAVATING A NEW TUNNEL alongside another structure, we sometimes want to know how much that structure will move so we can estimate the structural and serviceability impacts. For some structures, particularly other tunnels, a first approximation can be made by using simple analytical methods and case history data to predict the 'greenfield' horizontal ground movements and imposing these on the structure.

This article will focus on the prediction of horizontal ground movements at axis level of a tunnel using elastic and elastoplastic analytical solutions. In the next article we will look at case histories and empirical methods that are available.

Figure 2: Graphical definitions of uniform (left) and distortional (right) stress fields and deformations



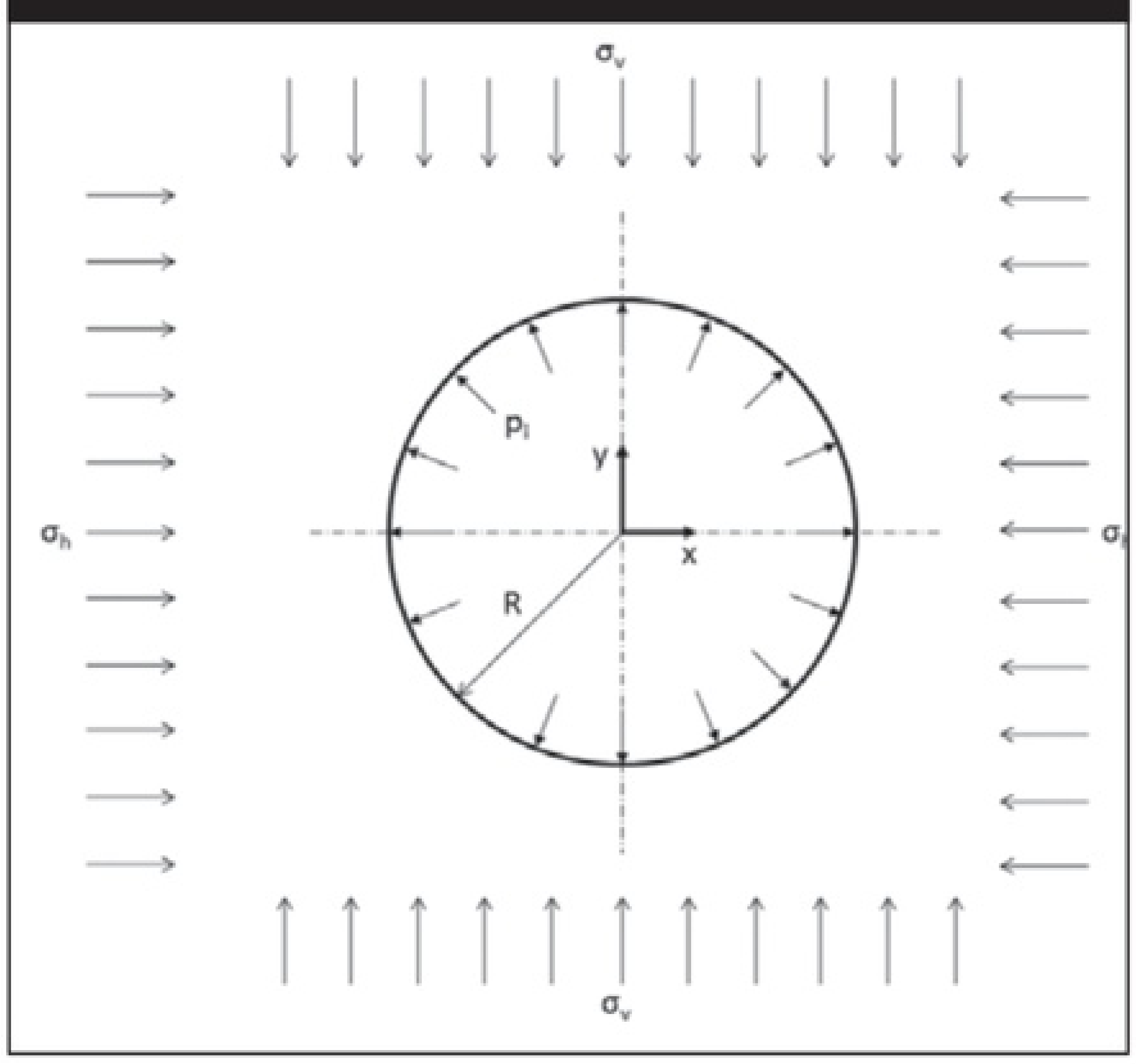
Elastic 2D plane strain solutions

IThe simplest model we can use to predict ground movements is a 2D plane strain model of a circular unlined tunnel in infinite elastic ground. The boundary conditions for such a model are shown in Figure 1, where σ_v is the vertical total stress, σ_h is the horizontal total stress, R is the tunnel radius and p_i is the internal support pressure.

'2D plane strain' means that strain is only allowed within the 2D plane. Another way of saying this is that no strain or displacement is allowed in the 3[rd] dimension, i.e. in the longitudinal tunnel direction. Therefore, the model behaves as though it is infinitely long and everything happens to the whole length of the tunnel at once.

In order to take account of 3D effects near to the face and the support to the ground the lining provides, a uniform internal pressure pi can be introduced in the model, as shown in Figure 1.

Figure 1: Boundary conditions for a simple 2D plane strain tunnel model



Another thing to notice is that the model shown in Figure 1 does not include the ground surface, and does not include gravity. There is a stress field applied, usually taken as the initial stress in the ground at the tunnel's axis level. This assumption obviously works better for deeper tunnels where the increase of stress with depth near to the tunnel is small relative to the stress level, and where the presence of the surface, which is free of shear stresses, has less effect on the ground movements.

The advantage of all these simplifications is that we end up with relatively simple equations, which were first discovered by Kirsch in 1898 (not to be confused with Kirsch's plane stress solution) and I will present below using notation from Pinto & Whittle (2014). First it is necessary to separate the problem into two modes of deformation: a uniform convergence and a purely distortional elliptical deformation, which are induced by a uniform stress field p_0 and a purely distortional stress field q_0 respectively, as follows:

$$p_0 = \sigma'_{v0}\frac{(1+K_0)}{2} + p_w$$

$$q_0 = \sigma'_{v0}\frac{(1-K_0)}{2}$$

Where K_0 is the coefficient of earth pressure at rest (= σ_h'/σ_v') and p_w is the pore pressure.

The uniform convergence u_ε and radial distortion u_δ at the tunnel boundary can be calculated as follows:

$$u_\varepsilon = \frac{(p_0 - p_i)R}{2G}$$

$$u_\delta = -\frac{q_0 R}{2G}(3-4\nu)$$

The definitions of p_0, q_0, u_ε and u_δ are shown graphically in Figure 2. R is the tunnel radius, G is the soil shear modulus and ν is the soil Poisson's ratio.

The displacement of any point (x,y) around the tunnel due to uniform convergence can be found using the following equations for displacement in the x direction u_x or y direction u_y:

$$u_x(x,y) = u_\varepsilon \frac{xR}{x^2+y^2}$$

$$u_y(x,y) = u_\varepsilon \frac{yR}{x^2+y^2}$$

The origin, where $x = 0$ and $y = 0$, is at the centre of the tunnel.

The displacement of any point (x,y) around the tunnel due to the elliptical distortion of the tunnel cavity can be found using the following equations:

$$u_x(x,y) = u_\delta \frac{R}{3-4\nu} x \frac{(3-4\nu)(x^2+y^2)^2-(3y^2-x^2)(x^2+y^2-R^2)}{(x^2+y^2)^3}$$

$$u_y(x,y) = -u_\delta \frac{R}{3-4\nu} y \frac{(3-4\nu)(x^2+y^2)^2-(3x^2-y^2)(x^2+y^2-R^2)}{(x^2+y^2)^3}$$

The final step is to add the deformations due to uniform convergence and the deformations due to distortion together to find the total effect. The principle of superposition applies because the ground is modelled as an elastic material.

Example:
A tunnel with radius R = 3m with initial vertical total stress σ_{v0} = 300 kPa, groundwater pressure p_w = 150 kPa, coefficient of earth pressure at rest K_0 = 0.7, soil shear modulus G = 20 MPa and Poisson's ratio ν = 0.5, representing an undrained soil. A uniform internal support pressure p_i = 100 kPa is applied.

Using the above equations we find p_0 = 277.5 kPa and q_0 = 22.5 kPa. Figure 3 shows the components of ground movements caused by uniform convergence and distortion. Note that because $K_0 < 1$, the tunnel squats so the distortional component, which has no net volume change, results in an outward movement of the ground at the axis level (as shown schematically in Figure 2). The total ground movements are found by adding the uniform convergence and distortional components.

Now we have these results we can find the effect of the new tunnel on an existing tunnel by assuming that the existing tunnel follows precisely the greenfield ground movements induced by the new tunnel.

If the existing tunnel were also 6 m diameter and its axis were 9 m from the axis of the new tunnel, then the nearest springline is 6 m from the new tunnel's axis and the furthest springline is 12 m from the new tunnel's axis, as shown in Figure 4. From the calculation for Figure 3, the horizontal ground movement at 6 m is 5.18 mm and at 12 m is 2.51 mm.

Using these values of horizontal movement we can calculate the radial distortion of the existing tunnel:

$$\delta = \frac{5.18-2.51}{2} = 1.335 \text{ mm}$$

We can then calculate the ovalisation of the existing tunnel:

$$\frac{1.335}{3000} = 0.045\%$$

This is a very small value, but the soil is elastic and we haven't yet included any plasticity. The radial distortion can be used to estimate the bending moment in the existing tunnel lining using Morgan's equation (Morgan, 1961):

$$M = \frac{3EI\delta}{R^2}$$

Where M is the moment, E and I are the Young's modulus and second moment of inertia of the tunnel lining, and R is the radius to the centroid of the tunnel lining. For a smoothbore (rectangular section) precast concrete lining, R would be the mean radius, but for standard rings or cast iron the centroid needs to be calculated.

So now we've got all these equations, let's do something with them. We can look at the effect of K_0, for instance, as shown in Figure 4. This shows that as K_0 increases, the horizontal movements increase. This is because the mean stress p_0 increases, increasing uniform convergence, and also because as K_0 increases the distortion of the tunnel changes from squatting when K_0<1 to no distortion at K_0=1, and then inverse-squatting when K_0 >1.

A quick note about axisymmetry
If the stress field were uniform, i.e. $\sigma_v=\sigma_h=\sigma_0$ (or K_0 = 1), then it is even simpler because the problem becomes axisymmetric. This means that the boundary of the tunnel will experience uniform convergence only and all ground movement vectors will be directed towards the centre of the tunnel and will vary only with radial distance r. If you remember that the equation of a circle centred on the origin is $r^2 = x^2 + y^2$, then you'll find the radial ground movements can be given by:

$$u_r = \frac{(\sigma_0 - p_i)R^2}{2Gr}$$

Note that this is really similar to the equation for uniform convergence at the tunnel boundary u_ε, except that it is multiplied by R/r. When $R = r$, i.e. at the tunnel boundary, they are the same.

As an approximation, and because the

Figure 3: Horizontal movements at axis level towards the tunnel using Kirsch's 2D plane strain elastic solution

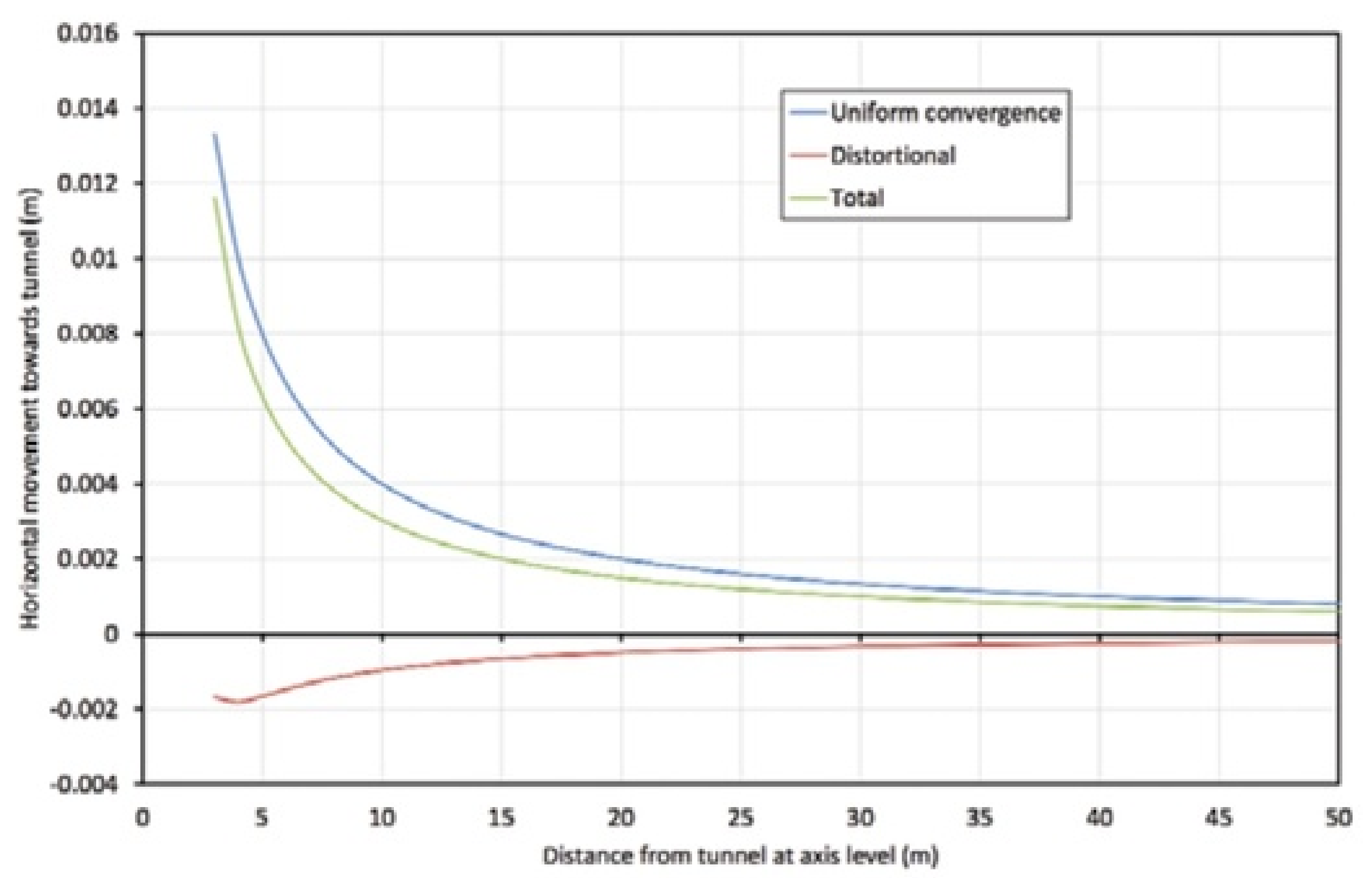


Figure 4: Cross-section showing a new 6 m tunnel constructed parallel to an existing 6 m tunnel at a spacing of 9 m

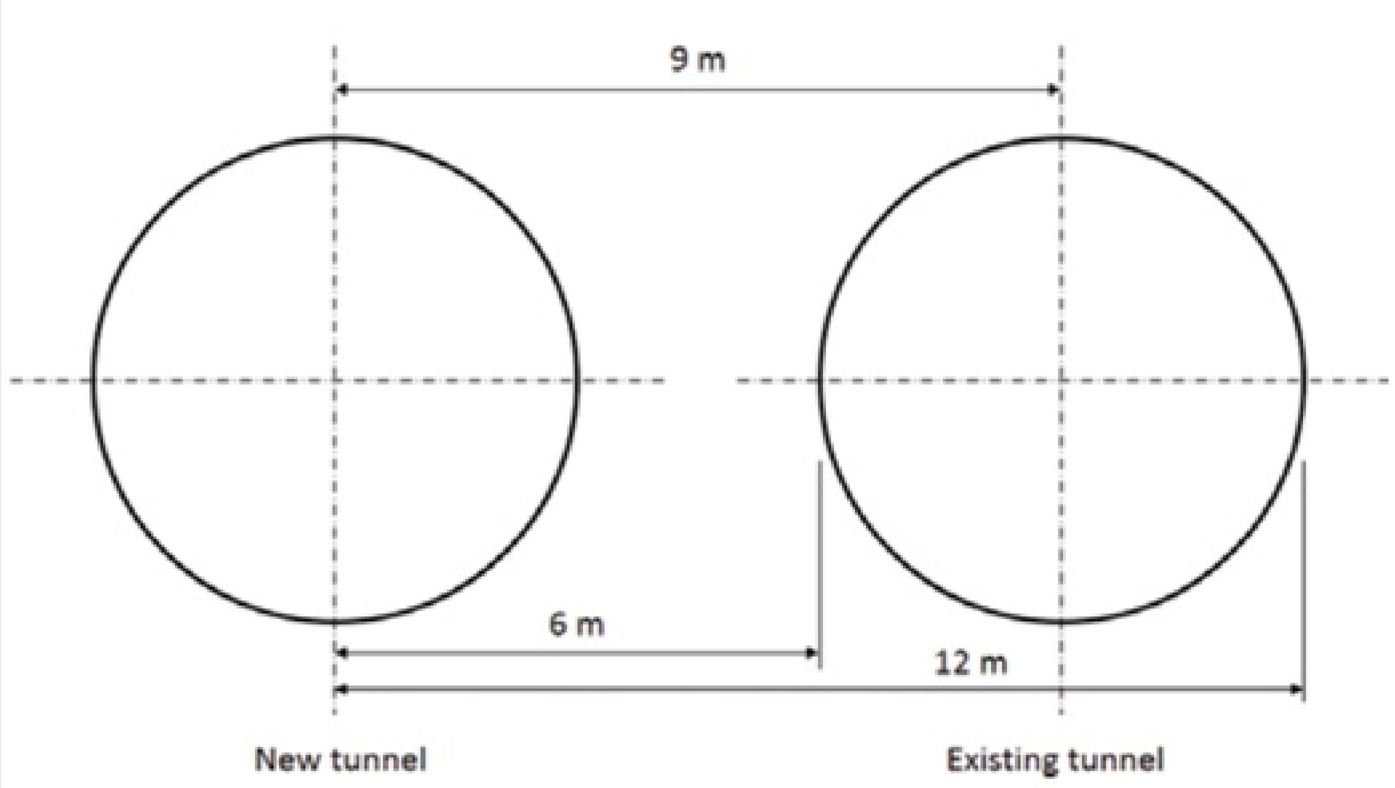


Figure 5: Effect of K_0 on horizontal ground movements at axis level - 2D elastic plane strain Kirsch model

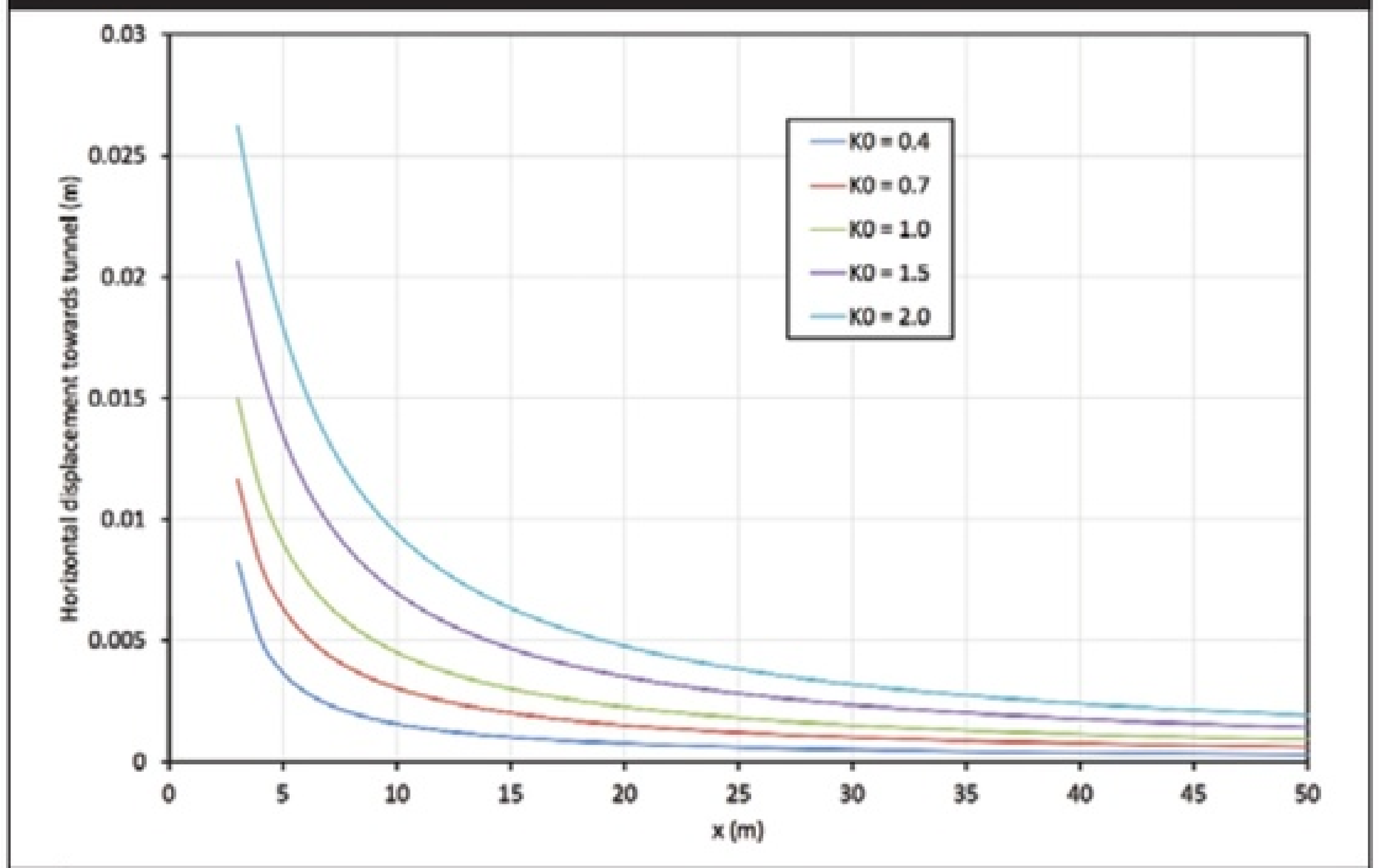
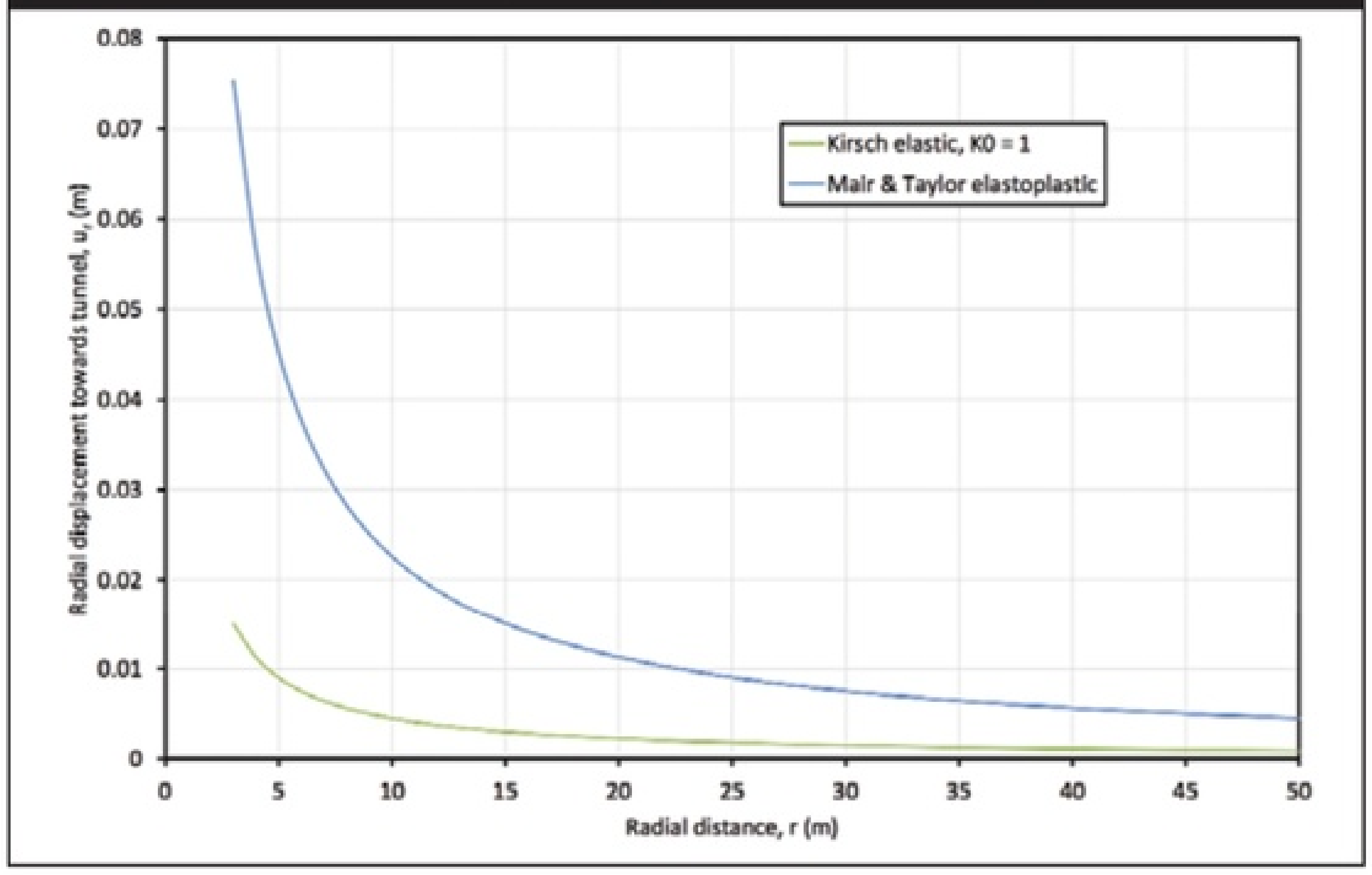


Figure 6: Comparison of horizontal ground movements at axis level from elastic and elastoplastic axisymmetric solutions for the example 6 m tunnel



equation is nice and simple, the axisymmetric approximation could be used to find the horizontal ground movements at axis level, but when $K_0 \neq 1$ it is difficult to know what value of σ_0 to use. More on that later.

Elastoplastic axisymmetric 2D models

Mair & Taylor (1993) presented an axisymmetric elastoplastic model for unloading of a cylindrical cavity in clay. This has the form:

$$u_r = \frac{S_u}{2G}\left(\frac{R^2}{r}\right) exp\left[\left(\frac{\sigma_0 - p_i}{S_u}\right) - 1\right]$$

Where S_u is the undrained shear strength of the clay.

The $(\sigma_0 - p_i)/S_u$ term is similar to the definition of stability number. This number is an indication of how close to failure the tunnel is. Mair & Taylor say that if $S_u = \sigma_0 - p_i$, then the soil will behave completely elastically. Try it and you'll see the equation becomes the same as the axisymmetric elastic equation in the previous section.

In order to make the problem simple enough to be easily solvable, Mair & Taylor have used axisymmetry. To apply a distortional stress field as well as a uniform convergence would be difficult because they couldn't be calculated separately and then superposed since the strains are not entirely elastic. Pinto & Whittle (2014) provide an analytical solution for ground movements that incorporates plasticity, but as far as I can tell this can only calculate surface settlements (or at least that is all the paper describes).

A comparison of the elastic and elastoplastic axisymmetric (i.e. $K_0 = 1$) solutions is shown in Figure 6. Introducing plasticity causes a large increase in ground movements in this case.

More complex elastic models

Sagaseta (1987) developed an elastic solution to take account of the presence of the ground surface. This solution was strain-controlled and so it only worked for incompressible soil (i.e. Poisson's ratio ν = 0.5). Verruijt & Booker (1996) adapted this so it could work for any value of Poisson's ratio.

Verruijt & Booker's solution applied a uniform radial ground loss to the boundary of the tunnel. Loganathan & Poulos (1998) improved Verruijt & Booker's solution by allowing more ground loss around the top of the tunnel, based on analysis of case histories and knowledge of typical construction processes. This resulted in a more realistic, narrower surface settlement trough than Verruijt & Booker's solution, though still wider than the case histories used for comparison in the paper.

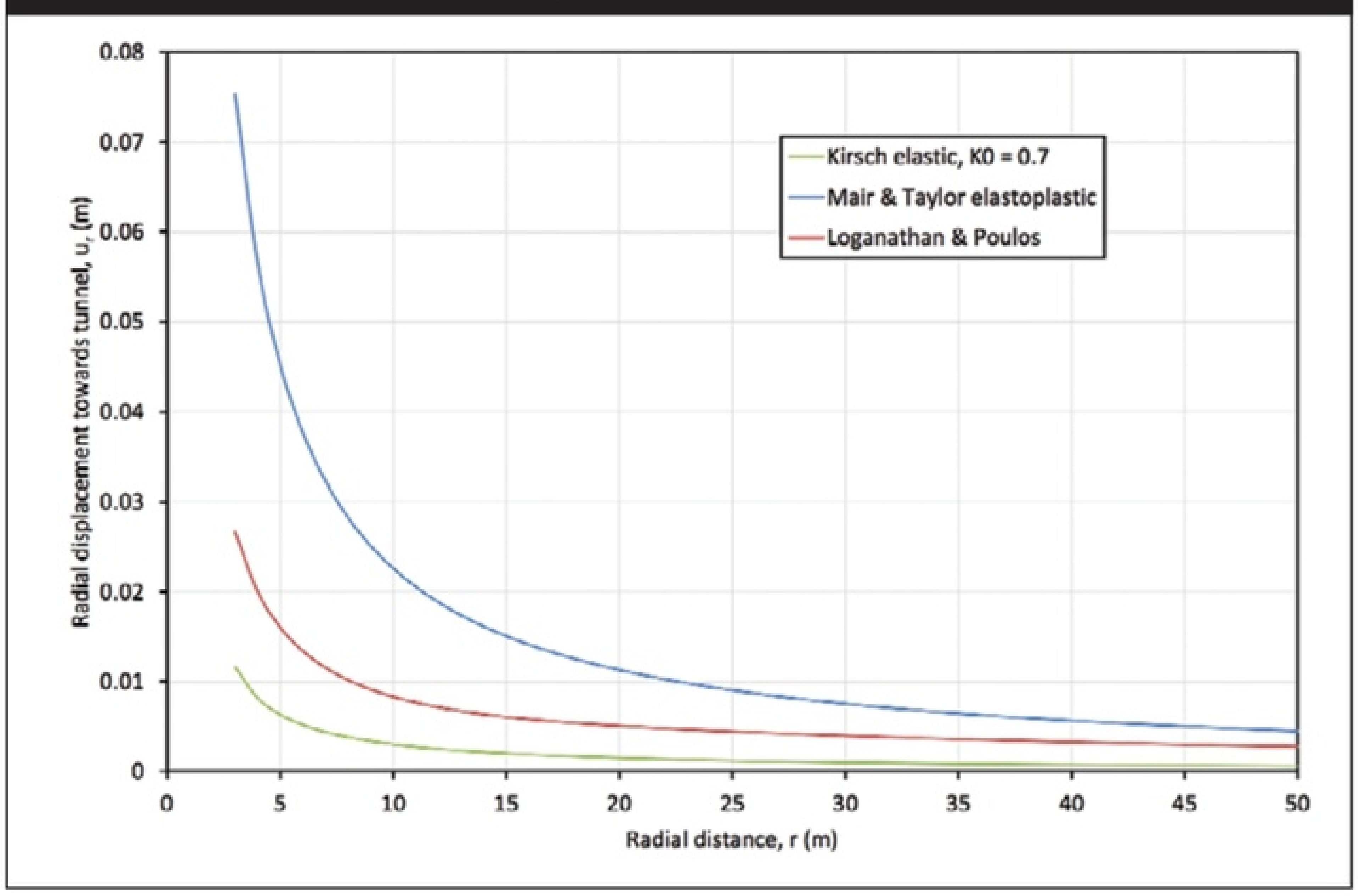
Loganathan & Poulos's solution has been compared to the Kirsch solution in Figure 7, assuming a depth to axis of 15 m for our 6 m example tunnel as described previously. Their equation for horizontal movements is:

$$u_x = -\varepsilon_0 R^2 x\left(\frac{1}{x^2+(H-z)^2}+\frac{3-4\nu}{x^2+(H+z)^2}-\frac{4z(z+H)}{\left(x^2+(H+z)^2\right)^2}\exp\left[-\left(\frac{1.38x^2}{(H+R)^2}+\frac{0.69z^2}{H^2}\right)\right]\right)$$

Where z is the depth from the surface to the point in question, H is the depth to tunnel axis, and ε_0 is the volume loss. For horizontal ground movements at axis level, $z = H$.

Note that the solution does not include any consideration of stress or stiffness, it is entirely displacement-driven. The value of

Figure 7: Horizontal ground movements from Loganathan & Poulos's solution compared with Kirsch elastic and Mair & Taylor elastoplastic exisymmetric solutions



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Pinto, F. & Whittle, A. J. (2014). Ground Movements due to Shallow Tunnels in Soft Ground I: Analytical Solutions. J. Geot. & Geoenv. Engrg ASCE 140 [page nos not available]
Sagaseta, C. (1987). Analysis of undrained soil deformation due to ground loss. Géotechnique 37(3), 301-320.
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Next time we will find out which are the best methods for making predictions.

K_0 is not an input, and distortion of the cavity is not calculated. The volume loss is an input rather than an output. For this case, the volume loss was set as 0.8875 %, in order to be the same as in the Kirsch solution.

Comparing Loganathan & Poulos's solution in Figure 7 to the Kirsch solution shows that modelling the presence of the surface and changing how the tunnel boundary deforms will make a difference to the predicted ground movement. The Kirsch solution allows the input of K0, so as K0 varies, the pattern of ground movements will change, whereas for Loganathan & Poulos's solution this is not the case, although it could be simulated indirectly by altering the distribution of displacement at the tunnel boundary.

Conclusions

Some relatively simple analytical solutions for horizontal ground movements at the axis level of a tunnel have been presented. In the next issue we will look at comparing these with empirical methods and case history data.

These analytical solutions are simplifications of the true situation, and as such they give us insights. For instance:

- Figure 3 showed us that distortion, as well as uniform convergence, are important in determining the ground movements around a tunnel.
- Figure 5 showed us that the worst case for horizontal movements at axis level is when K0 is higher.
- Figure 6 showed us that close to a tunnel in clay, plasticity will become more and more important as the ratio of in situ stress to undrained shear strength increases (or in other words, as the stability number increases).
- Figure 7 showed us that modelling the presence of the surface and changing how the tunnel boundary deforms will make a difference to the predicted ground movements.