PROBABILISTIC METHODS

In this article, Dr Benoît Jones looks at probabilistic methods for designing tunnels.

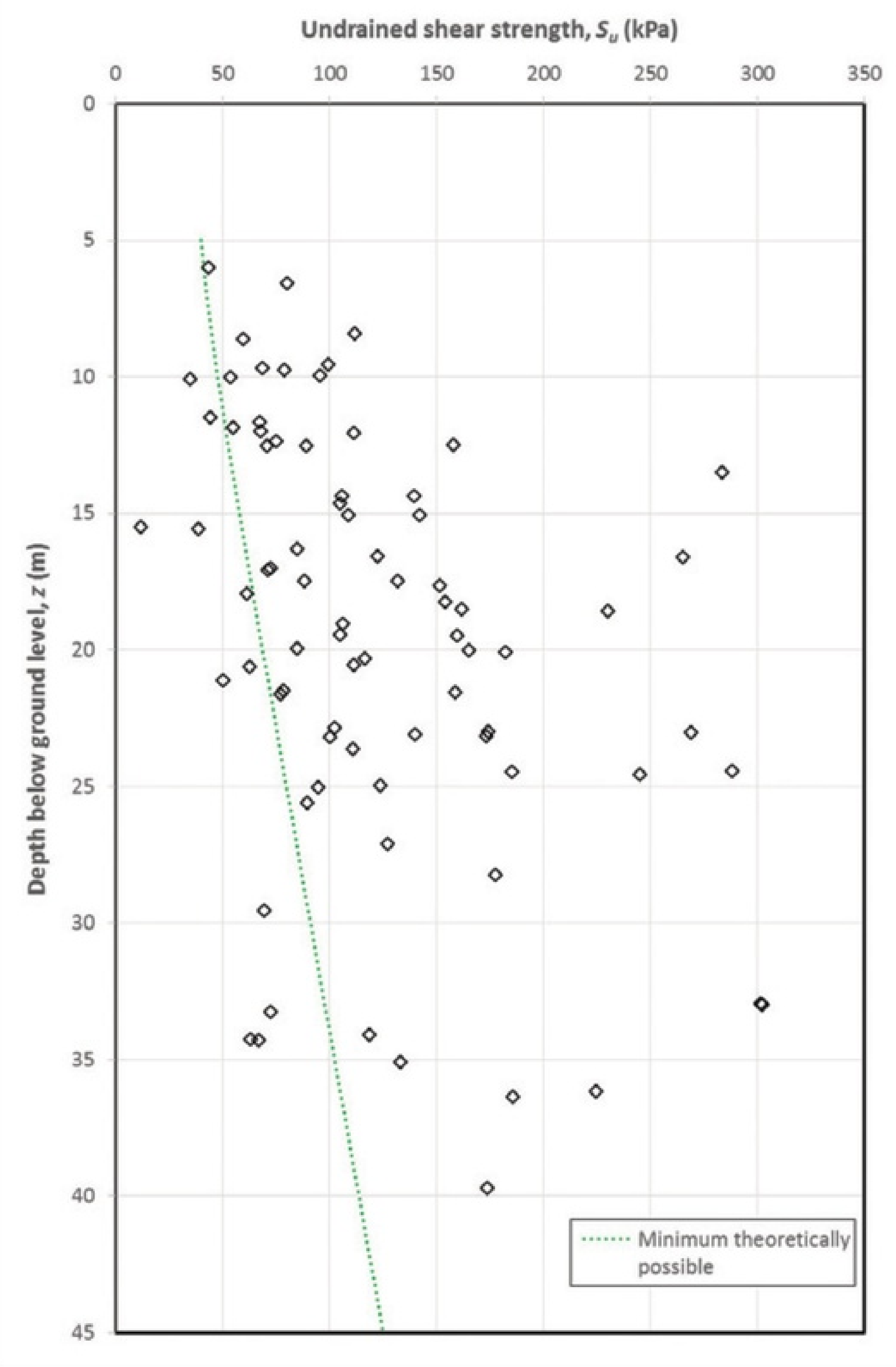
WHEN DESIGNING A TUNNEL, we start with input parameters, put them into a model, then a prediction comes out the other end. Sometimes the prediction is not what we want, so we change the design and try again, so it is often an iterative process.

The usual input parameters are:

- Geotechnical parameters
- Groundwater pressures
- Geometry of everything: tunnel, ground surface, strata levels (or dips and strikes of strata if not horizontal), etc.
- Lining material properties and section dimensions
- Imposed loads, e.g. surcharge, support pressures, compensation grouting, M&E equipment loads etc.

First we'll take a look at some real data to see why geotechnical design is so difficult – because selecting the values of geotechnical parameters to use in design requires an in-depth knowledge of statistics and a lot of skill and judgement. In recent years, some people have tried to take a probabilistic approach instead, and this is what the focus of the rest of this article will be.

Figure 1: Undrained shear strength vs depth



Real ground properties

In any particular stratum, we assume that the ground has either constant properties or that these vary with depth. However, the real world is not like that. The real world is like this (Figure 1):

In Figure 1, which is typical of good quality site investigation data, one can see that there may be a trend of increasing S_u with depth, but there is a considerable amount of scatter. It is not obvious where we should place a linear trendline that represents the mean values.

A theoretical minimum possible value of S_u can be introduced, based on a minimum historic overburden of 5m, assuming the unit weight of the clay is 20 kN/m³ and assuming the groundwater table is at the surface both historically and at present, and assuming a ratio $(S_u/\sigma_v')_{nc}$ = 0.23 when normally consolidated, using the following equation:

$$\left(\frac{S_u}{\sigma_v'}\right) = \left(\frac{S_u}{\sigma_v'}\right)_{nc} \times OCR^{0.8}$$

This theoretical minimum possible value is shown in Figure 1 as the dotted line.

If we ignore values below the theoretical minimum, assuming that these samples must have been disturbed (Bond & Harris, 2008:146), and if we assume the data is normally distributed, then after some linear regression, one can find the linear trend line of the mean (the best fit line) as:

$$S_u(\text{kPa}) = 50 + 4.4z$$

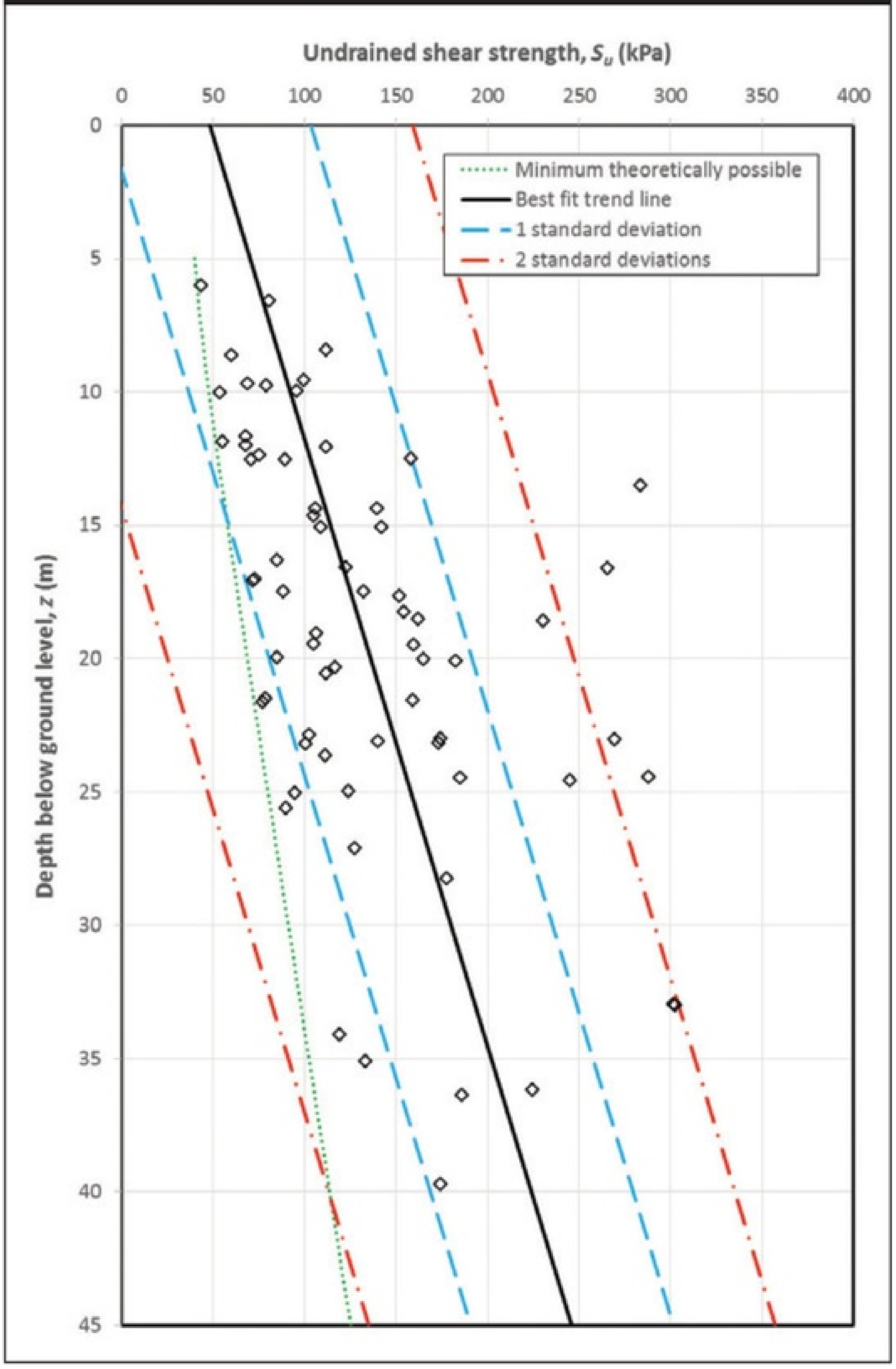
This best fit trend line is shown in Figure 2. The standard deviation may also be calculated, and for this data it is 55.5 kPa. This is also shown on Figure 2

It would probably be better to use a lognormal distribution in this case, since it seems there is more scatter to the right of the best fit line than to the left. In these situations the mean and standard deviation may tend to increase as the sample size increases when a normal distribution is used, whereas the statistics of a lognormal distribution remain constant and will model the data more closely.

Variability of input parameters

When we design underground structures, to allow for uncertainty

Figure 2: Statistical interpretation of undrained shear strength data assuming a normal distribution



the values of the soil or rock parameters we choose are, according to Eurocode 7, "a cautious estimate of the value affecting the occurrence of the limit state" (EN 1997-1, 2.4.5.2(2)P). These cautious values are termed 'characteristic values'. We are going to assume for now that we are considering a limit state governed by a large zone of ground, for instance global stability or structural design of the tunnel lining.

In order to estimate the characteristic value, we need to know the statistics, but we also need to know the confidence level. In order to assess confidence, we can use Student's t-distribution. Based on the number of samples, we can find the 5% fractile of the mean value at any depth using multivariate statistics, and these are the characteristic values. I won't go into the details here; very good guidance with examples can be found in Bond & Harris (2008).

By taking a cautious estimate of the mean as the characteristic value, we are implicitly assuming that all the samples we have tested in the site investigation are represented in the zone of ground we are considering, i.e. that the 'scale of fluctuation' of the parameters is much smaller than the zone of ground being considered.

This single characteristic value, or single linear relationship as in our example, is then used in design. All the site investigation data is boiled down to a single value or equation using engineering judgement and/or statistical methods. Although the design is probably safe, we actually don't know the probability of failure.

An alternative to this approach is to use probabilistic methods, which retain the variability of the input parameters. The most straightforward method would be to randomly sample values of all the input parameters that are subject to variation. One might assume that the surface and strata levels are known and constant, so it is usually only the soil parameters and sometimes the structural materials such as concrete, that are sampled. It is important that the random sampling is weighted such that after a large number of samples are taken, the distribution of the sampled input parameters is the same as the actual distribution of the soil parameter.

Each time a set of random values are sampled, an analysis is performed. After a large number of samplings and analyses, the results will be found to converge on a probability distribution. This distribution can be used to ensure the design has an acceptably low probability of failure. This is known as a Monte Carlo analysis. To achieve convergence can take up to tens of thousands or hundreds of thousands of cycles, depending on the number of variables and the acceptable error. If one had to manually (or even automatically) run a finite element analysis each time, this would be quite a big job.

Mollon et al. (2013) and Miro et al. (2015) both got around this drawback of the Monte Carlo method by using a computationally

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inexpensive analytical surrogate model instead of the full 3D finite element model. The 3D numerical model of the tunnel was used to produce an analytical response surface to characterise the output, which was then used in the Monte Carlo analysis. This sounds a bit like telling it what response you expect, then surprise surprise you get the response you expected, but it is actually a bit more rigorous than that. Both Miro et al. and Mollon et al. only modelled a single homogeneous stratum of soil.

There are other ways to make Monte Carlo analysis much more efficient, and lately these have been applied to tunnel designs. Svoboda & Hilar (2015) used a method called Latin Hypercube Sampling. This is a method that provides a set of samples that are carefully weighted so that they will reproduce the same mean and standard deviation as the input parameter, as well as the correlations between input parameters. Thus, a much reduced number of samples is required and hence a much reduced number of analyses.

Nasekhian et al. (2012) used the Point Estimate Method and the Random Set Method on a tunnel problem. These methods also dramatically decrease the number of simulations required, while maintaining acceptable accuracy of the output probability distribution function. Again, the problem was simplified by assuming homogeneous ground. The model was 2D, but one of the variables was the relaxation factor, which was used to take account of the 3D effect.

Spatial variability

As we have seen, the standard approach to selecting a characteristic value of a geotechnical parameter for a limit state dependent on a large zone of ground assumes that the variability within that zone of ground is similar to the variability in the site investigation data. In effect, the scale of fluctuation of geotechnical parameters is assumed to be very small compared to the limit state considered (Hicks & Nuttall, 2012). Only one value is assigned to the ground in the design model and the ground is assumed to be homogeneous. Similarly, the major drawback with the probabilistic methods described so far is that in every simulation the soil or rock is assumed homogeneous. This in essence assumes that the scale of fluctuation of parameters is much larger than the zone of ground governing the limit state.

There are other ways to make Monte Carlo analysis much more efficient, and lately these have been applied to tunnel designs.

We know from site investigation data (for example, Figure 1) that the ground is rarely homogeneous, and that there is spatial variation of parameters within it (Phoon & Kulhaway, 1999a & 1999b). Phoon & Kulhawy (1999a) quote a range of values of vertical scale of fluctuation between 1m and 6m and horizontal scale of fluctuation between 3m and 80m for strength parameters in a variety of different soil types. Therefore it usually cannot be considered to be either much larger or much smaller than the zone of ground affecting the occurrence of the limit state.

So the ground is heterogeneous. It has weaker areas and stronger areas, softer areas and stiffer areas. As it is loaded, the real stress paths are far more complex than we can imagine. The question then is: does it matter? Does heterogeneous ground behave differently to homogeneous ground? Or does heterogeneity somehow average out?

Spatial variation can be modelled by using a Monte Carlo method to assign random values of the parameters to different areas within a model. This is sometimes called the 'random field method'. The size of the areas is key – an 'auto-correlation distance' is defined, which is related to the scale of fluctuation, over which parameters can be considered to remain constant.

For slope stability, Hicks & Nuttall (2012) and Griffiths & Fenton (2007) found that the mean factor of safety was lower when spatial variation was modelled, as failure surfaces followed a path of least resistance. This may be due to the different scales of problem and different scales of fluctuation modelled, and also how the areas modelled lined up with failure surfaces. They used the random field method to apply values of parameters to a finite element mesh that were spatially correlated, that is, where the distance between midpoints of the finite elements determined how much the parameters could vary. This is sometimes referred to as the 'random finite element method' or 'RFEM' (Fenton & Griffiths, 2007). Papaioannou et al. (2009) also found that ignoring spatial variation led to an overestimate of probability of failure, this time for a rock tunnel with 1100m of overburden.

Conclusions

The standard Eurocode 7 approach to design assumes that the scale of fluctuation of geotechnical parameters is much smaller than the zone of ground affecting the occurrence of the limit state. The design is safe but the probability of failure is unknown.

Probabilistic methods are becoming more accessible as clever methods to reduce the number of simulations are introduced, computer power increases and commercially-available software begins to incorporate it as an option. These methods allow calculation of the probability of failure, but ignore spatial variability. As we have seen, probabilistic methods assume that the scale of fluctuation of geotechnical parameters is much larger than the zone of ground affecting the occurrence of the limit state.

Real soil or rock is heterogeneous, with a scale of fluctuation that is rarely much larger or much smaller than the zone of ground affecting the occurrence of the limit state (Phoon & Kulhawy, 1999a). Studies comparing RFEM with probabilistic methods have found that the probability of failure is different. Therefore, modelling this heterogeneity may be of interest. Applying heterogeneity to the design model using coefficient of variation and scale of fluctuation in a random field approach may be where the future lies.

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